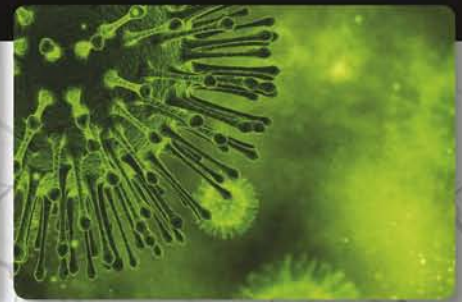


Applied Mathematics

For the Managerial, Life, and Social Sciences
Seventh Edition



TIJAN

A Sampling of New Applications

Drawn from diverse fields of interest and situations that occur in the real world



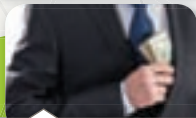
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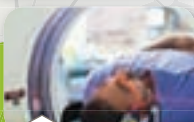
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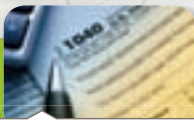
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EDITION

7

APPLIED MATHEMATICS

FOR THE MANAGERIAL, LIFE,
AND SOCIAL SCIENCES

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Soo T. Tan

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TO PAT, BILL, AND MICHAEL



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SOO T. TAN received his S.B. degree from Massachusetts Institute of Technology, his M.S. degree from the University of Wisconsin–Madison, and his Ph.D. from the University of California at Los Angeles. He has published numerous papers in optimal control theory, numerical analysis, and mathematics of finance. He is also the author of a series of calculus textbooks.

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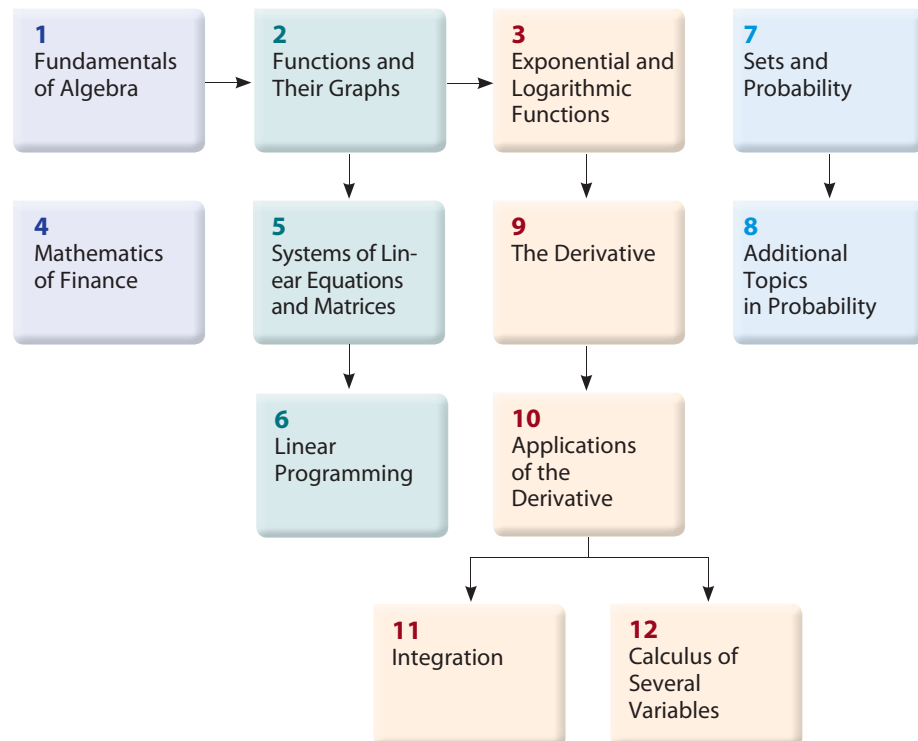
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PREFACE

Math plays a vital role in our increasingly complex daily life. *Applied Mathematics for the Managerial, Life, and Social Sciences* attempts to illustrate this point with its applied approach to mathematics. Students have a much greater appreciation of the material if the applications are drawn from their fields of interest and from situations that occur in the real world. This is one reason you will see so many exercises in my texts that are modeled on data gathered from newspapers, magazines, journals, and other media. In addition, many students come into this course with some degree of apprehension. For this reason, I have adopted an intuitive approach in which I try to introduce each abstract mathematical concept through an example drawn from a common life experience. Once the idea has been conveyed, I then proceed to make it precise, thereby ensuring that no mathematical rigor is lost in this intuitive treatment of the subject.

This text offers more than enough material for a two-semester or three-semester course. The following chart on chapter dependency is provided to help the instructor design a course that is most suitable for the intended audience.



Note that in this text exponential and logarithmic functions are treated early.

The Approach

Presentation

Consistent with my intuitive approach, I state the results informally. However, I have taken special care to ensure that mathematical precision and accuracy are not compromised.

Problem-Solving Emphasis

Special emphasis is placed on helping students formulate, solve, and interpret the results of applied problems. Because students often have difficulty setting up and solving word problems, extra care has been taken to help them master these skills.

- Very early in the text, students are given guidelines for setting up word problems (see Section 2.7). This is followed by numerous examples and exercises to help students master this skill.
- One entire section is devoted to modeling and setting up linear programming problems (see Section 6.2).
- First, in Section 10.4, techniques of calculus are used to solve optimization problems in which the function to be optimized is given. Later, in Section 10.5, optimization problems that require the additional step of formulating the problem are treated.

Absolute Extrema on a Closed Interval

As the preceding examples show, a continuous function defined on an arbitrary interval does not always have an absolute maximum or an absolute minimum. But an important case arises often in practical applications in which both the absolute maximum and the absolute minimum of a function are guaranteed to exist. This occurs when a continuous function is defined on a *closed* interval.

Before stating this important result formally, let's look at a real-life example. The graph of the function f in Figure 60 shows the average price, $f(t)$, in dollars, of domestic airfares by days before flight. The domain of f is the closed interval $[-210, -1]$, where -210 is interpreted as 210 days before flight and -1 is interpreted as the day before flight.

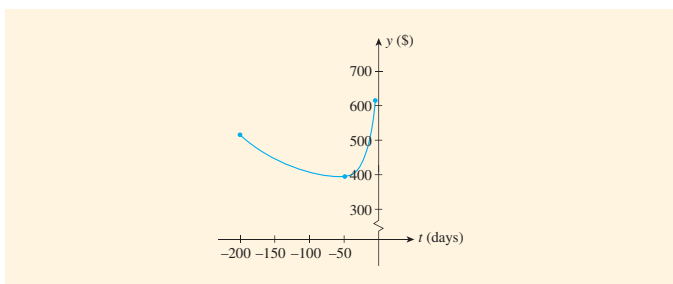


FIGURE 60
Average price before flight
Source: Cheapair.com.

Observe that f attains the minimum value of 395 when $t = -49$ and the maximum value of 614 when $t = -1$. This result tells us that the best time to book a domestic flight is seven weeks in advance and the worst day to book a domestic flight is the day before the flight. Probably most surprising of all, booking too early can be almost as expensive as booking too late. Note that the function f is continuous on a closed interval. For such functions, we have the following theorem.

Motivation

Illustrating the practical value of mathematics in applied areas is an objective of my approach. Concepts are introduced with concrete, real-life examples wherever appropriate. These examples and other applications have been chosen from current topics and issues in the media and serve to answer a question often posed by students: “What will I ever use this for?” In this new edition, for example, the concept of finding the absolute extrema over a closed interval is introduced as shown at left.

Modeling

One important skill that every student should acquire is the ability to translate a real-life problem into a mathematical model. In Section 2.7, the modeling process is discussed, and students are asked to use models (functions) constructed from real-life data to answer questions. Additionally, students get hands-on experience constructing these models in the Using Technology sections.

New to this Edition

The focus of this revision has been the continued emphasis on illustrating the mathematical concepts in *Applied Mathematics* by using more real-life applications that are relevant to the everyday life of students and to their fields of study in the managerial, life, and social sciences. Over 300 new applications have been added in the examples and exercises. A sampling of these new applications is provided on the inside front cover pages.

Many of the exercise sets have been revamped. In particular, the exercise sets were restructured to follow more closely the order of the presentation of the material in each section and to progress more evenly from easier to more difficult problems in both the rote and applied sections of each exercise set. Additional concept questions, rote exercises, and true-or-false questions were also included.

More Specific Content Changes

Chapters 1 and 2 In Section 1.1, the introduction to real numbers was rewritten for clarity. New rote, concept, and application exercises were also added. In Section 2.2, newly added Example 8b–c illustrates how to determine whether a point lies on a line. Also, a new application, *Smokers in the United States*, has been added to the Self-Check Exercises 2.2. The U.S. federal budget deficit graphs that are used as motivation to introduce “The Algebra of Functions” in Section 2.4 have been updated to reflect the current deficit situation. In Section 2.5, Applied Example 1, *Erosion of the Middle Class*, has been added. Students are shown how the model for this application is constructed in newly added Section 2.8, *The Method of Least Squares*. New models and graphs for the *Global Warming*, *Social Security Trust Fund Assets*, and *Driving Costs* applications appear in Section 2.7.

Chapters 3 and 4 New application exercises were added to Chapter 3. These include *The Decline of American Idol*, *Renewable Energy*, *Bank Failures*, and *Obesity Over Time*, among others. In Chapter 4, interest-rate problems throughout the chapter were revised to reflect the current interest-rate environment. Also, in Section 4.3, two new exercises were added illustrating the new Ability-to-Repay rules for mortgages adopted by the Consumer Financial Protection Bureau in response to the financial crisis.

Chapters 5 and 6 New application exercises have been added, and many examples and exercises have been updated. These include *Using Digital Technology*, *Terrorism Polls*, *Model Investment Portfolios*, *Comparative Shopping*, and *Airline Flight Scheduling*, among others. Also, in Section 6.1, newly added Example 6 illustrates how to determine whether a point lies in a feasible set of inequalities. This is followed by a new application, *A Production Problem*, in which students are shown how they can use a solution set for a given system of inequalities (restrictions) to determine whether certain production goals can be met. Also, in Section 6.1, Exercise 44, we see how the solution of a system of linear equations is obtained by looking at a system of inequalities.

Chapters 7 and 8 These chapters deal with the calculations of probabilities and data analysis. The emphasis here is placed on providing data from marketing, economic, consumer, and scientific surveys that are relevant, current, and of interest to students to motivate the mathematical concepts presented. Topics in these exercises and examples include the greatest challenge on starting a new job, the number of years it will take for people to fully recover from the Great Recession, the most common cause of on-the-job distractions, the number of social media accounts that individuals have, and the financial hardship caused by gas prices.

Also, several new examples were added in these chapters. Example 13 in Section 7.1, *Cyber Privacy*, illustrates set operations. Example 5 in Section 7.2 illustrates how the solution of a system of linear equations can sometimes be used to help draw a Venn diagram. In Section 8.2, Example 8 illustrates the difference between mutually exclusive and independent events. Also, Example 12, *Predicting Travel Weather*, illustrates the calculation of the probability of independent events. Example 8, *Commuting Times*, in Section 8.5, illustrates the calculation of expected value for grouped data; and Example 4 in Section 8.6, *Married Males*, illustrates the calculation of standard deviation for grouped data. Using Technology Section 8.4 was expanded to include an example (Example 3, *Time Use of College Students*) and exercises illustrating how Excel can be used to create pie charts.

Chapters 9 and 10 A wealth of new application exercises has been added throughout these chapters. A new model and graph, *Income of American Households*, have been added as an introduction to exponential models in Section 9.7. This is followed by an analysis of the function describing the graph in the Using Technology exercises. A new subsection on relative rates of change and a new application, *Inflation*, have been added to Section 9.8. In Section 10.1, the U.S. budget deficit (surplus) graph that is used to introduce relative extrema has been updated. In the Section 10.4 exercises, the absolute extrema for this same deficit function are found in Problem 72. Also, in Section 10.4, a new application, *Average Fare Before a Flight*, has been added to introduce the concept of absolute extrema on a closed interval.

Chapters 11 and 12 Many new applications have been added to these chapters. The intuitive discussion of area and the definite integral at the beginning of Section 11.3, as illustrated by the total daily petroleum consumption of a New England state, is given a firmer mathematical footing at the end of the section by demonstrating that the petroleum consumption of the state is indeed given by the area under a curve. The 3-D art in the text and exercises in Chapter 12 has been further enhanced.

Features

Motivating Applications

Many new applied examples and exercises have been added in the Seventh Edition. Among the topics of the new applications are Cyber Monday, family insurance coverage, leveraged return, credit card debt, tax refund fraud, salaries of married women, and online video advertising.

Portfolios

These interviews share the varied experiences of professionals who use mathematics in the workplace. Among those included are a Vice President of Wealth Management Advisor and an associate at The Mason Group.

Explore and Discuss

These optional questions can be discussed in class or assigned as homework. They generally require more thought and effort than the usual exercises. They may also be used to add a writing component to the class or as team projects.

Real-World Connections

20. ONLINE VIDEO ADVERTISING Although still a small percentage of all online advertising, online video advertising is growing. The following table gives the projected spending on Web video advertising (in billions of dollars) through 2016:

Year	2011	2012	2013	2014	2015	2016
Spending, y	2.0	3.1	4.5	6.3	7.8	9.3

- Letting $x = 0$ denote 2011, find an equation of the least-squares line for these data.
- Use the result of part (a) to estimate the projected rate of growth of video advertising from 2011 through 2016.

Source: eMarketer.

PORTFOLIO Robert H. Mason



TITLE Vice President, Wealth Management Advisor
INSTITUTION The Mason Group

The Mason Group—a team of financial advisors at a major wire house firm—acts as an interface between clients and investment markets. To meet the needs of our private clients, it is important for us to maintain constant contact with them, adjusting their investments when the markets and allocations change and when the clients' goals change.

We often help our clients determine whether their various expected sources of income in retirement will provide them with their desired retirement lifestyle. To begin, we determine the after-tax funds needed in retirement in today's dollars in consultation with the client. Using simple arithmetic, we then look at the duration and amount of their various income flows and their current portfolio allocations (stocks, bonds, cash, etc.). Once we have made this assessment, we take into consideration future possible allocations to arrive at a range of probabilities for portfolio valuation for all years up to and including retirement.

For example, we might tell a client that, on the basis of their given investment plan, there is an 80–95% probability that the expected value of his or her portfolio will increase from \$1 million to about \$1.5 million by the time the client turns 90; a 50–80% chance that it will increase from \$1 million to around \$2 million; and a 30–50% chance that it will increase to approximately \$2.8 million.

To arrive at these probabilities, we use a deterministic model that assumes that a constant annual rate of return is applied to the portfolio every year of the analysis. We also use probabilistic modeling, taking into account various factors such as economic conditions, the allocation of assets, and market volatility. By using these techniques we are able to arrive at a confidence level, without making any guarantees, that our clients will be able to attain the lifestyle in retirement they desire.



Stockter Grubb (inset) © iStockPhoto.com/Eugene Choi

Explore and Discuss

The average price of gasoline at the pump over a 3-month period, during which there was a temporary shortage of oil, is described by the function f defined on the interval $[0, 3]$. During the first month, the price was increasing at an increasing rate. Starting with the second month, the good news was that the rate of increase was slowing down, although the price of gas was still increasing. This pattern continued until the end of the second month. The price of gas peaked at $t = 2$ and began to fall at an increasing rate until $t = 3$.

- Describe the signs of $f'(t)$ and $f''(t)$ over each of the intervals $(0, 1)$, $(1, 2)$, and $(2, 3)$.
- Make a sketch showing a plausible graph of f over $[0, 3]$.

Explorations and Technology

Exploring with Technology

These optional discussions appear throughout the main body of the text and serve to enhance the student's understanding of the concepts and theory presented. Often the solution of an example in the text is augmented with a graphical or numerical solution.

Exploring with TECHNOLOGY

Refer to Example 4. Suppose Marcus wished to know how much he would have in his IRA at any time in the future, not just at the beginning of 2014, as you were asked to compute in the example.

- Using Formula (18) and the relevant data from Example 4, show that the required amount at any time x (x measured in years, $x > 0$) is given by

$$A = f(x) = 40,000(e^{0.05x} - 1)$$

- Use a graphing utility to plot the graph of f , using the viewing window $[0, 30] \times [0, 200,000]$.
- Using **ZOOM** and **TRACE**, or using the function evaluation capability of your graphing utility, use the result of part 2 to verify the result obtained in Example 4. Comment on the advantage of the mathematical model found in part 1.

Using Technology

Written in the traditional example-exercise format, these optional sections show how to use the graphing calculator and Microsoft Excel 2010 as a tool to solve problems. Illustrations showing graphing calculator screens and spreadsheets are used extensively. In keeping with the theme of motivation through real-life examples, many sourced applications are included.

A *How-To Technology Index* is included at the back of the book for easy reference to Using Technology examples.



APPLIED EXAMPLE 3 Time Use of College Students Use the data given in Table 1 to construct a pie chart.

TABLE T1	
Time Used on an Average Weekday for Full-Time University and College Students	
Time Use	Time (in hours)
Sleeping	8.5
Leisure and sports	3.7
Working and related activities	2.9
Educational activities	3.3
Eating and drinking	1.0
Grooming	0.7
Traveling	1.5
Other	2.4

Source: Bureau of Labor Statistics.

Solution

We begin by entering the information from Table T1 in Columns A and B on a spreadsheet. Then follow these steps:

- Step 1 First, highlight the data in cells A2:A9 and B2:B9 as shown in Figure T4.

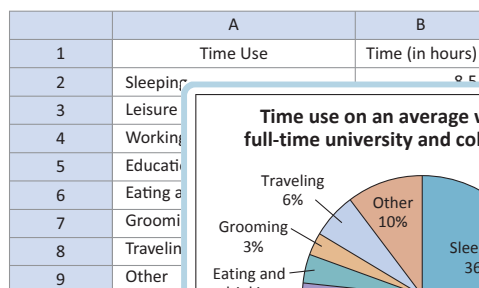


FIGURE T4 Completed spreadsheet

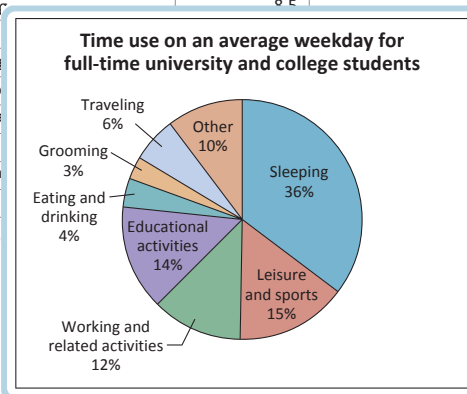


FIGURE T5 The pie chart describing the data in Table T1

Concept Building and Critical Thinking

Self-Check Exercises

Offering students immediate feedback on key concepts, these exercises begin each end-of-section exercise set and contain both rote and word problems (applications). Fully worked-out solutions can be found at the end of each exercise section. If students get stuck while solving these problems, they can get immediate help before attempting to solve the homework exercises. Applications have been included here because students often need extra practice with setting up and solving these problems.

Concept Questions

Designed to test students' understanding of the basic concepts discussed in the section, these questions encourage students to explain learned concepts in their own words.

Exercises

Each section contains an ample set of exercises of a routine computational nature followed by an extensive set of modern application exercises.

9.3 Self-Check Exercises

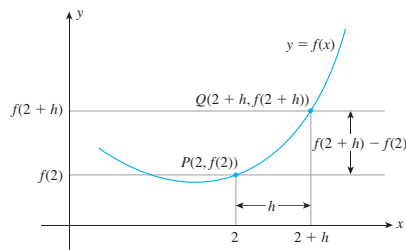
- Let $f(x) = -x^2 - 2x + 3$.
 - Find the derivative f' of f , using the definition of the derivative.
 - Find the slope of the tangent line to the graph of f at the point $(0, 3)$.
 - Find the rate of change of f when $x = 0$.
 - Find an equation of the tangent line to the graph of f at the point $(0, 3)$.
 - Sketch the graph of f and the tangent line to the curve at the point $(0, 3)$.
- BANK LOSSES** The losses (in millions of dollars) due to bad loans extended chiefly in agriculture, real estate, shipping, and energy by the Franklin Bank are estimated to be

$$A = f(t) = -t^2 + 10t + 30 \quad (0 \leq t \leq 10)$$
 where t is the time in years ($t = 0$ corresponds to the beginning of 2007). How fast were the losses mounting at the beginning of 2010? At the beginning of 2012? At the beginning of 2014?

Solutions to Self-Check Exercises 9.3 can be found on page 646.

9.3 Concept Questions

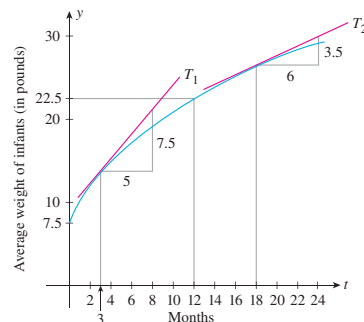
For Questions 1 and 2, refer to the following figure.



- Let $P(2, f(2))$ and $Q(2+h, f(2+h))$ be points on the graph of a function f .
 - Find an expression for the slope of the secant line passing through P and Q .
 - Find an expression for the slope of the tangent line passing through P .
- Refer to Question 1.
 - Find an expression for the average rate of change of f over the interval $[2, 2+h]$.
 - Find an expression for the instantaneous rate of change of f at 2.
 - Compare your answers for parts (a) and (b) with those of Question 1.

9.3 Exercises

- AVERAGE WEIGHT OF AN INFANT** The following graph shows the weight measurements of the average infant from the time of birth ($t = 0$) through age 2 ($t = 24$). By computing the slopes of the respective tangent lines, estimate the rate of change of the average infant's weight when $t = 3$ and when $t = 18$. What is the average rate of change in the average infant's weight over the first year of life?

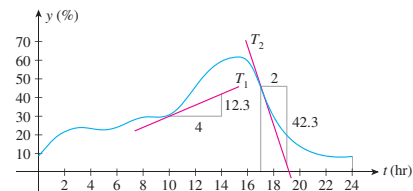


- FORESTRY** The following graph shows the volume of wood produced in a single-species forest. Here, $f(t)$ is measured in cubic meters per hectare, and t is measured in years. By computing the slopes of the respective tangent lines, estimate the rate at which the wood grown is changing at the beginning of year 10 and at the beginning of year 30.

Source: The Random House Encyclopedia.

- TV-VIEWING PATTERNS** The following graph shows the percentage of U.S. households watching television during a 24-hr period on a weekday ($t = 0$ corresponds to 6 A.M.). By computing the slopes of the respective tangent lines, estimate the rate of change of the percent of households watching television at 4 P.M. and 11 P.M.

Source: A. C. Nielsen Company.



- CROP YIELD** Productivity and yield of cultivated crops are often reduced by insect pests. The following graph shows the relationship between the yield of a certain crop, $f(x)$, as a function of the density of aphids x . (Aphids are small insects that suck plant juices.) Here, $f(x)$ is measured in kilograms per 4000 square meters, and x is measured in hundreds of aphids per bean stem. By computing the slopes of the respective tangent lines, estimate the rate of change of the crop yield with respect to the density of aphids when that density is 200 aphids/bean stem and when it is 800 aphids/bean stem.

Source: The Random House Encyclopedia.

Review and Study Tools

Summary of Principal Formulas and Terms

Each review section begins with the Summary, which highlights the important equations and terms, with page numbers given for quick review.

CHAPTER 2 Summary of Principal Formulas and Terms		
FORMULAS		
1. Slope of a line	$m = \frac{y_2 - y_1}{x_2 - x_1}$	
2. Equation of a vertical line	$x = a$	
3. Equation of a horizontal line	$y = b$	
4. Point-slope form of the equation of a line	$y - y_1 = m(x - x_1)$	
5. Slope-intercept form of the equation of a line	$y = mx + b$	
6. General equation of a line	$Ax + By + C = 0$	
TERMS		
Cartesian coordinate system (72)	dependent variable (94)	profit function (119)
ordered pair (72)	graph of a function (95)	break-even point (121)
coordinates (72)	piecewise-defined function (97)	quadratic function (131)
parallel lines (76)	graph of an equation (98)	demand function (134)
perpendicular lines (80)	Vertical Line Test (98)	supply function (135)
function (92)	composite function (110)	market equilibrium (136)
domain (92)	linear function (116)	polynomial function (143)
range (92)	total cost function (118)	rational function (146)
independent variable (94)	revenue function (119)	power function (146)

Concept Review Questions

These questions give students a chance to check their knowledge of the basic definitions and concepts given in each chapter.

CHAPTER 2 Concept Review Questions	
<p>Fill in the blanks.</p> <ol style="list-style-type: none"> A point in the plane can be represented uniquely by a/an _____ pair of numbers. The first number of the pair is called the _____, and the second number of the pair is called the _____. <ol style="list-style-type: none"> The point $P(a, 0)$ lies on the _____-axis, and the point $P(0, b)$ lies on the _____-axis. If the point $P(a, b)$ lies in the fourth quadrant, then the point $P(-a, b)$ lies in the _____ quadrant. <ol style="list-style-type: none"> If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are any two distinct points on a nonvertical line L, then the slope of L is $m =$ _____. The slope of a vertical line is _____. The slope of a horizontal line is _____. The slope of a line that slants upward from left to right is _____. If L_1 and L_2 are nonvertical lines with slopes m_1 and m_2, respectively, then L_1 is parallel to L_2 if and only if _____ and L_1 is perpendicular to L_2 if and only if _____. 	<p>x takes on all possible values in A is called the _____ of f. The range of f is contained in the set _____.</p> <ol style="list-style-type: none"> The graph of a function is the set of all points (x, y) in the xy-plane such that x is in the _____ of f and $y =$ _____. The Vertical Line Test states that a curve in the xy-plane is the graph of a function $y = f(x)$ if and only if each _____ line intersects it in at most one _____. If f and g are functions with domains A and B, respectively, then (a) $(f \pm g)(x) =$ _____, (b) $(fg)(x) =$ _____, and (c) $\left(\frac{f}{g}\right)(x) =$ _____. The domain of $f + g$ is _____. The domain of $\frac{f}{g}$ is _____ with the additional condition that $g(x)$ is never _____. The composition of g and f is the function with rule $(g \circ f)(x) =$ _____. Its domain is the set of all x in the domain of _____ such that _____ lies in the domain of _____.

Review Exercises

Offering a solid review of the chapter material, the Review Exercises contain routine computational exercises followed by applied problems.

CHAPTER 2 Review Exercises	
<p>In Exercises 1–6, find an equation of the line L that passes through the point $(-2, 4)$ and satisfies the given condition.</p> <ol style="list-style-type: none"> L is a vertical line. L is a horizontal line. L passes through the point $(3, \frac{2}{3})$. The x-intercept of L is 3. L is parallel to the line $5x - 2y = 6$. L is perpendicular to the line $4x + 3y = 6$. Find an equation of the line with slope $-\frac{1}{2}$ and y-intercept -3. 	<ol style="list-style-type: none"> Find the slope and y-intercept of the line with equation $3x - 5y = 6$. Find an equation of the line passing through the point $(2, 3)$ and parallel to the line with equation $3x + 4y - 8 = 0$. Find an equation of the line passing through the point $(-1, 3)$ and parallel to the line joining the points $(-3, 4)$ and $(2, 1)$. Find an equation of the line passing through the point $(-2, -4)$ that is perpendicular to the line with equation $2x - 3y - 24 = 0$.

Before Moving On . . .

Found at the end of each chapter review, these exercises give students a chance to determine whether they have mastered the basic computational skills developed in the chapter.

CHAPTER 2 Before Moving On . . .

1. Find an equation of the line that passes through $(-1, -2)$ and $(4, 5)$.
2. Find an equation of the line that has slope $-\frac{1}{3}$ and y -intercept $\frac{4}{3}$.
3. Let

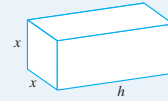
$$f(x) = \begin{cases} -2x + 1 & \text{if } -1 \leq x < 0 \\ x^2 + 2 & \text{if } 0 \leq x \leq 2 \end{cases}$$

Find (a) $f(-1)$, (b) $f(0)$, and (c) $f(\frac{3}{2})$.

4. Let $f(x) = \frac{1}{x+1}$ and $g(x) = x^2 + 1$. Find the rules for (a) $f + g$, (b) fg , (c) $f \circ g$, and (d) $g \circ f$.

5. **POSTAL REGULATIONS** Postal regulations specify that a parcel sent by parcel post may have a combined length and girth of no more than 108 in. Suppose a rectangular package that has a square cross section of x in. \times x in. is to have a combined length and girth of exactly 108 in. Find a function in terms of x giving the volume of the package.

Hint: The length plus the girth is $4x + h$ (see the accompanying figure).

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S. T. Tan

1

FUNDAMENTALS OF ALGEBRA

THIS CHAPTER CONTAINS a brief review of the algebra you will use in this course. In the process of solving many practical problems, you will need to solve algebraic equations. You will also need to simplify algebraic expressions. This chapter also contains a short review of inequalities and absolute value; their uses range from describing the domains of functions to formulating applied problems.

How much money is needed to purchase at least 100,000 shares of the Starr Communications Company? Corbyco, a giant conglomerate, wishes to purchase a minimum of 100,000 shares of the company. In Example 8, page 60, you will see how Corbyco's management determines how much money they will need for the acquisition.



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1.1 Real Numbers

The Real Numbers

We use *real numbers* every day to describe various quantities such as temperature, salary, annual percentage rate, shoe size, and grade point average. Some of the symbols we use to represent real numbers are

$$3, -17, \sqrt{2}, 0.666\dots, 113, 3.9, 0.12875$$

To construct the real numbers, we start with the **natural numbers** (also called counting numbers)

$$1, 2, 3, 4, \dots$$

and adjoin other numbers to them. The **whole numbers**

$$0, 1, 2, 3, \dots$$

are obtained by adjoining the single number 0 to the natural numbers. By adjoining the negatives of the natural numbers to the whole numbers, we obtain the **integers**:

$$\dots, -3, -2, -1, 0, 1, 2, 3, \dots$$

Next, we consider the **rational numbers**, numbers of the form $\frac{a}{b}$, where a and b are integers, with $b \neq 0$. Observe that every integer is a rational number, since each integer may be written in the form $\frac{a}{b}$, with $b = 1$. For example, the integer 6 may be written in the form $\frac{6}{1}$. But a rational number is not necessarily an integer, since fractions such as $\frac{1}{2}$ and $\frac{23}{25}$ are not integers.

Finally, we obtain the real numbers by adjoining the rational numbers to the **irrational numbers**—numbers that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers ($b \neq 0$). Examples of irrational numbers are $\sqrt{2}$, $\sqrt{3}$, π , and so on. Thus, the **real numbers** comprise all rational numbers and irrational numbers. (See Figure 1.)

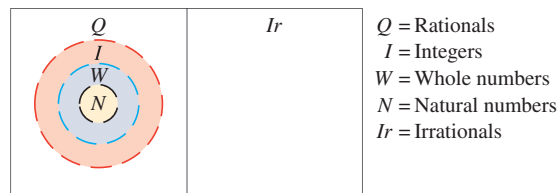


FIGURE 1
The real numbers consist of the rational numbers and the irrational numbers.

Representing Real Numbers as Decimals

Every real number can be written as a decimal. A rational number can be represented as either a repeating decimal or a terminating decimal. For example, $\frac{2}{3}$ is represented by the repeating decimal

$$0.6666666\dots \quad \text{Repeating decimal; note that the integer 6 repeats.}$$

and may also be written as $0.\overline{6}$, where the bar above the 6 indicates that the 6 repeats indefinitely. The number $\frac{1}{2}$ is represented by the terminating decimal

$$0.5 \quad \text{Terminating decimal}$$

When an irrational number is represented as a decimal, it neither terminates nor repeats. For example,

$$\sqrt{2} = 1.41421\dots \quad \text{and} \quad \pi = 3.14159\dots$$

Table 1 summarizes this classification of real numbers.

TABLE 1			
The Real Numbers			
Type	Description	Examples	Decimal Representation
Natural numbers	Counting numbers	1, 2, 3, . . .	Terminating decimals
Whole numbers	Counting numbers and 0	0, 1, 2, 3, . . .	Terminating decimals
Integers	Natural numbers, their negatives, and 0	. . . , -3, -2, -1, 0, 1, 2, 3, . . .	Terminating decimals
Rational numbers	Numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$	-3, $-\frac{3}{4}$, $-0.22\bar{2}$, $0, \frac{5}{6}$, 2, 4.3111	Terminating or repeating decimals
Irrational numbers	Numbers that cannot be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$	$\sqrt{2}$, $\sqrt{3}$, π , 1.414213. . . , 1.732050. . .	Nonterminating, non-repeating decimals
Real numbers	Rational and irrational numbers	All of the above	All types of decimals

Representing Real Numbers on a Number Line

Real numbers may be represented geometrically by points on a line. This *real number*, or *coordinate*, *line* is constructed as follows: Arbitrarily select a point on a straight line to represent the number 0. This point is called the *origin*. If the line is horizontal, then choose a point at a convenient distance to the right of the origin to represent the number 1. This determines the scale for the number line.

The point representing each positive real number x lies x units to the right of 0, and the point representing each negative real number x lies $-x$ units to the left of 0. Thus, real numbers may be represented by points on a line in such a way that corresponding to each real number there is exactly one point on a line, and vice versa (Figure 2).

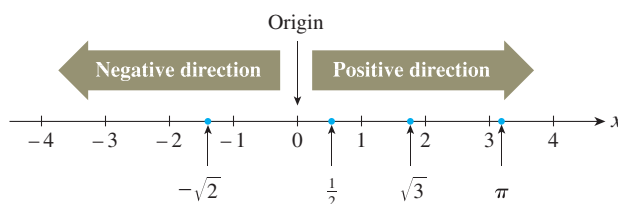


FIGURE 2
The real number line

Operations with Real Numbers

Two real numbers may be combined to obtain a real number. The operation of addition, written $+$, enables us to combine any two numbers a and b to obtain their *sum*, denoted by $a + b$. Another operation, multiplication, written \cdot , enables us to combine any two real numbers a and b to form their product, the number $a \cdot b$ (more simply written ab). These two operations are subject to the rules of operation given in Table 2.

The operation of subtraction is defined in terms of addition. Thus,

$$a + (-b)$$

where $-b$ is the additive inverse of b , may be written in the more familiar form $a - b$, and we say that b is subtracted from a . Similarly, the operation of division is defined in terms of multiplication. Recall that the multiplicative inverse of a nonzero real number b is $\frac{1}{b}$, also written b^{-1} . Then,

$$a\left(\frac{1}{b}\right)$$

is written $\frac{a}{b}$, and we say that a is divided by b . Thus, $4(\frac{1}{3}) = \frac{4}{3}$. Remember, zero does not have a multiplicative inverse, since division by zero is not defined.

TABLE 2

Rules of Operation for Real Numbers

Rule		Example
Addition		
1. $a + b = b + a$	Commutative law of addition	$2 + 3 = 3 + 2$
2. $a + (b + c) = (a + b) + c$	Associative law of addition	$4 + (2 + 3) = (4 + 2) + 3$
3. $a + 0 = a$	Identity law of addition	$6 + 0 = 6$
4. $a + (-a) = 0$	Inverse law of addition	$5 + (-5) = 0$
Multiplication		
1. $ab = ba$	Commutative law of multiplication	$3 \cdot 2 = 2 \cdot 3$
2. $a(bc) = (ab)c$	Associative law of multiplication	$4(3 \cdot 2) = (4 \cdot 3)2$
3. $a \cdot 1 = 1 \cdot a$	Identity law of multiplication	$4 \cdot 1 = 1 \cdot 4$
4. $a\left(\frac{1}{a}\right) = 1 \quad (a \neq 0)$	Inverse law of multiplication	$3\left(\frac{1}{3}\right) = 1$
Addition and multiplication		
1. $a(b + c) = ab + ac$	Distributive law for multiplication with respect to addition	$3(4 + 5) = 3 \cdot 4 + 3 \cdot 5$

Do the operations of associativity and commutativity hold for subtraction and division? Looking first at associativity, we see that the answer is no, since

$$a - (b - c) \neq (a - b) - c \quad 7 - (4 - 2) \neq (7 - 4) - 2, \text{ or } 5 \neq 1$$

and

$$a \div (b \div c) \neq (a \div b) \div c \quad 8 \div (4 \div 2) \neq (8 \div 4) \div 2, \text{ or } 4 \neq 1$$

Similarly, commutativity does not hold because

$$a - b \neq b - a \quad 7 - 4 \neq 4 - 7, \text{ or } 3 \neq -3$$

and

$$a \div b \neq b \div a \quad 8 \div 4 \neq 4 \div 8, \text{ or } 2 \neq \frac{1}{2}$$

EXAMPLE 1 State the real number property that justifies each statement.

Statement	Property
a. $4 + (x - 2) = 4 + (-2 + x)$	Commutative law of addition
b. $(a + 2b) + c = a + (2b + c)$	Associative law of addition
c. $x(y - z + 2) = (y - z + 2)x$	Commutative law of multiplication
d. $4(xy^2) = (4x)y^2$	Associative law of multiplication
e. $x(y - 2) = xy - 2x$	Distributive law for multiplication under addition

Using the properties of real numbers listed earlier, we can derive all other algebraic properties of real numbers. Some of the more important properties are given in Tables 3–5.

TABLE 3

Properties of Negatives

Property	Example
1. $-(-a) = a$	$-(-6) = 6$
2. $(-a)b = -(ab) = a(-b)$	$(-3)4 = -(3 \cdot 4) = 3(-4)$
3. $(-a)(-b) = ab$	$(-3)(-4) = 3 \cdot 4$
4. $(-1)a = -a$	$(-1)5 = -5$

TABLE 4

Properties Involving Zero

Property

- $a \cdot 0 = 0$
- If $ab = 0$, then either $a = 0$ or $b = 0$, or both a and $b = 0$.

TABLE 5

Properties of Quotients

Property

Example

- | | |
|---|---|
| 1. $\frac{a}{b} = \frac{c}{d}$ if $ad = bc$ | $\frac{3}{4} = \frac{9}{12}$ because $3 \cdot 12 = 9 \cdot 4$ |
| 2. $\frac{ca}{cb} = \frac{a}{b}$ | $\frac{4 \cdot 3}{4 \cdot 8} = \frac{3}{8}$ |
| 3. $\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$ | $\frac{4}{-3} = \frac{-4}{3} = -\frac{4}{3}$ |
| 4. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ | $\frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}$ |
| 5. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ | $\frac{3}{4} \div \frac{5}{2} = \frac{3}{4} \cdot \frac{2}{5} = \frac{3}{10}$ |
| 6. $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ | $\frac{3}{4} + \frac{5}{2} = \frac{3 \cdot 2 + 4 \cdot 5}{8} = \frac{13}{4}$ |
| 7. $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$ | $\frac{3}{4} - \frac{5}{2} = \frac{3 \cdot 2 - 4 \cdot 5}{8} = -\frac{7}{4}$ |

Note Here and in the rest of this book, we assume that all variables are restricted so that the denominator of a quotient is not equal to zero. ■

EXAMPLE 2 State the real number property that justifies each statement.

Statement

Property

a. $-(-4) = 4$

Property 1 of negatives

b. If $(4x - 1)(x + 3) = 0$, then $x = \frac{1}{4}$ or $x = -3$.

Property 2 of zero properties

c. $\frac{(x - 1)(x + 1)}{(x - 1)(x - 3)} = \frac{x + 1}{x - 3}$

Property 2 of quotients

d. $\frac{x - 1}{y} \div \frac{y + 1}{x} = \frac{x - 1}{y} \cdot \frac{x}{y + 1} = \frac{x(x - 1)}{y(y + 1)}$

Property 5 of quotients

e. $\frac{x}{y} + \frac{x}{y + 1} = \frac{x(y + 1) + xy}{y(y + 1)}$

Property 6 of quotients

$$= \frac{xy + x + xy}{y(y + 1)} = \frac{2xy + x}{y(y + 1)}$$

Distributive law ■

1.1 Self-Check Exercises

State the property (or properties) that justify each statement.

1. $(3v + 2) - w = 3v + (2 - w)$

2. $(3s)(4t) = 3[s(4t)]$

3. $-(-s + t) = s - t$

4. $\frac{2}{-(u - v)} = -\frac{2}{u - v}$

Solutions to Self-Check Exercises 1.1 can be found on page 7.

1.1 Concept Questions

- Give an example of each of the following:
 - A natural number
 - A whole number that is not a natural number
 - A rational number that is not an integer
 - An irrational number
- Give an example of each of the following:
 - A rational number that is represented by a terminating decimal
 - A rational number that is represented by a repeating decimal
 - An irrational number represented by a decimal
- The associative law of addition states that $a + (b + c) = \underline{\hspace{2cm}}$.
 - The distributive law states that $ab + ac = \underline{\hspace{2cm}}$.
- Does the operation of associativity hold for subtraction? For division? Explain.
 - Does zero have a multiplicative inverse? Explain.
- What can you say about a and b if $ab \neq 0$? How about a , b , and c if $abc \neq 0$?

1.1 Exercises

In Exercises 1–10, classify the number as to type. (For example, $\frac{1}{2}$ is rational and real, whereas $\sqrt{5}$ is irrational and real.)

- -3
- -420
- $\frac{3}{8}$
- $-\frac{4}{125}$
- $\sqrt{11}$
- $-\sqrt{5}$
- $\frac{\pi}{2}$
- $\frac{2}{\pi}$
- 2.421
- $2.71828\dots$

In Exercises 11–16, indicate whether the statement is true or false.

- Every integer is a whole number.
- Every integer is a rational number.
- Every natural number is an integer.
- Every rational number is a real number.
- Every natural number is an irrational number.
- Every irrational number is a real number.

In Exercises 17–36, state the real number property that justifies the statement.

- $(2x + y) + z = z + (2x + y)$
- $3x + (2y + z) = (3x + 2y) + z$
- $u(3v + w) = (3v + w)u$
- $a^2(b^2c) = (a^2b^2)c$
- $u(2v + w) = 2uv + uw$
- $(2u + v)w = 2uw + vw$
- $(2x + 3y) + (x + 4y) = 2x + [3y + (x + 4y)]$
- $(a + 2b)(a - 3b) = a(a - 3b) + 2b(a - 3b)$
- $a - [-(c + d)] = a + (c + d)$

$$26. -(2x + y)[-(3x + 2y)] = (2x + y)(3x + 2y)$$

$$27. 0(2a + 3b) = 0$$

$$28. \text{If } (x - y)(x + y) = 0, \text{ then } x = y \text{ or } x = -y.$$

$$29. \text{If } (x - 2)(2x + 5) = 0, \text{ then } x = 2 \text{ or } x = -\frac{5}{2}.$$

$$30. \text{If } x(2x - 9) = 0, \text{ then } x = 0 \text{ or } x = \frac{9}{2}.$$

$$31. \frac{(x + 1)(x - 3)}{(2x + 1)(x - 3)} = \frac{x + 1}{2x + 1}$$

$$32. \frac{(2x + 1)(x + 3)}{(2x - 1)(x + 3)} = \frac{2x + 1}{2x - 1}$$

$$33. \frac{a + b}{b} \div \frac{a - b}{ab} = \frac{a(a + b)}{a - b}$$

$$34. \frac{x + 2y}{3x + y} \div \frac{x}{6x + 2y} = \frac{x + 2y}{3x + y} \cdot \frac{2(3x + y)}{x} = \frac{2(x + 2y)}{x}$$

$$35. \frac{a}{b + c} + \frac{c}{b} = \frac{ab + bc + c^2}{b(b + c)}$$

$$36. \frac{x + y}{x + 1} - \frac{y}{x} = \frac{x^2 - y}{x(x + 1)}$$

In Exercises 37–42, indicate whether the statement is true or false.

- If $ab = 1$, then $a = 1$ or $b = 1$.
- If $ab = 0$ and $a \neq 0$, then $b = 0$.
- $a - b = b - a$
- $a \div b = b \div a$
- $(a - b) - c = a - (b - c)$
- $a \div (b \div c) = (a \div b) \div c$

1.1 Solutions to Self-Check Exercises

1. Associative law of addition: $a + (b + c) = (a + b) + c$
2. Associative law of multiplication: $a(bc) = (ab)c$
3. Distributive law for multiplication: $a(b + c) = ab + ac$
Properties 1 and 4 of negatives: $-(-a) = a$; $(-1)a = -a$
4. Property 3 of quotients: $\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$

1.2 Polynomials

Exponents

Expressions such as 2^5 , $(-3)^2$, and $(\frac{1}{4})^4$ are exponential expressions. More generally, if n is a natural number and a is a real number, then a^n represents the product of the real number a and itself n times.

Exponential Notation

If a is a real number and n is a natural number, then

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}} \quad 3^4 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{4 \text{ factors}}$$

The natural number n is called the **exponent**, and the real number a is called the **base**.

EXAMPLE 1

- $4^4 = (4)(4)(4)(4) = 256$
- $(-5)^3 = (-5)(-5)(-5) = -125$
- $(\frac{1}{2})^3 = (\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = \frac{1}{8}$
- $(-\frac{1}{3})^2 = (-\frac{1}{3})(-\frac{1}{3}) = \frac{1}{9}$

When we evaluate expressions such as $3^2 \cdot 3^3$, we use the following property of exponents to write the product in exponential form.

Property 1

If m and n are natural numbers and a is any real number, then

$$a^m \cdot a^n = a^{m+n} \quad 3^2 \cdot 3^3 = 3^{2+3} = 3^5$$

To verify that Property 1 follows from the definition of an exponential expression, we note that the total number of factors in the exponential expression

$$a^m \cdot a^n = \underbrace{a \cdot a \cdot a \cdots a}_{m \text{ factors}} \cdot \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$$

is $m + n$.

EXAMPLE 2

- a. $3^2 \cdot 3^3 = 3^{2+3} = 3^5 = 243$
 b. $(-2)^2 \cdot (-2)^5 = (-2)^{2+5} = (-2)^7 = -128$
 c. $(3x) \cdot (3x)^3 = (3x)^{1+3} = (3x)^4 = 81x^4$

 Be careful to apply the exponent to the indicated base only. Note that

$$4 \cdot x^2 = 4x^2 \neq (4x)^2 = 4^2 \cdot x^2 = 16x^2$$

↙ The exponent applies to $4x$.
↕
↘ The exponent applies only to the base x .

and

$$-3^2 = -9 \neq (-3)^2 = 9$$

↙ The exponent applies to -3 .
↕
↘ The exponent applies only to the base 3 .

Polynomials

Recall that a *variable* is a letter that is used to represent any element of a given set. Unless specified otherwise, variables in this text will represent real numbers. Sometimes physical considerations impose restrictions on the values a variable may assume. For example, if the variable x denotes the number of LCD television sets sold daily in an appliance store, then x must be a nonnegative integer. At other times, restrictions must be imposed on x in order for an expression to make sense. For example, in the expression $\frac{1}{x+2}$, x cannot take on the value -2 , since division by 0 is not permitted. We call the set of all real numbers that a variable is allowed to assume the *domain of the variable*.

In contrast to a variable, a *constant* is a fixed number or letter whose value remains fixed throughout a particular discussion. For example, in the expression $\frac{1}{2}gt^2$, which gives the distance in feet covered by a free-falling body near the surface of the earth, t seconds from rest, the letter g represents the constant of acceleration due to gravity (approximately 32 feet/second/second), whereas the letter t is a variable with domain consisting of nonnegative real numbers.

By combining constants and variables through addition, subtraction, multiplication, division, exponentiation, and root extraction, we obtain *algebraic expressions*. Examples of algebraic expressions are

$$3x - 4y \quad 2x^2 - y + \frac{1}{xy} \quad \frac{ax - b}{1 - x^2} \quad \frac{3xy^{-2} + \pi}{x^2 + y^2 + z^2}$$

where a and b are constants and x , y , and z are variables. Intimidating as some of these expressions might be, remember that they are just real numbers. For example, if $x = 1$ and $y = 4$, then the second expression represents the number

$$2(1)^2 - 4 + \frac{1}{(1)(4)}$$

or $-\frac{7}{4}$, obtained by replacing x and y in the expression by the appropriate values.

Polynomials are an important class of algebraic expressions. The simplest polynomials are those involving *one* variable.

Polynomial in One Variable

A **polynomial** in x is an expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where n is a nonnegative integer and a_0, a_1, \dots, a_n are real numbers, with $a_n \neq 0$.

The expressions $a_k x^k$ in the sum are called the *terms* of the polynomial. The numbers a_0, a_1, \dots, a_n are called the *coefficients* of $1, x, x^2, \dots, x^n$, respectively. The coefficient a_n of x^n (the highest power in x) is called the *leading coefficient* of the polynomial. The nonnegative integer n gives the *degree* of the polynomial. For example, consider the polynomial

$$-2x^5 + 8x^3 - 6x^2 + 3x + 1$$

1. The terms of the polynomial are $-2x^5, 8x^3, -6x^2, 3x$, and 1 .
2. The coefficients of $1, x, x^2, x^3, x^4$, and x^5 are $1, 3, -6, 8, 0$, and -2 , respectively.
3. The leading coefficient of the polynomial is -2 .
4. The degree of the polynomial is 5 .

A polynomial that has just one term (such as $2x^3$) is called a *monomial*; a polynomial that has exactly two terms (such as $x^3 + x$) is called a *binomial*; and a polynomial that has only three terms (such as $-2x^3 + x - 8$) is called a *trinomial*. Also, a polynomial consisting of one (constant) term a_0 (such as the monomial -8) is called a *constant polynomial*. Observe that the degree of a constant polynomial a_0 , with $a_0 \neq 0$, is 0 because we can write $a_0 = a_0 x^0$ and see that $n = 0$ in this situation. If all the coefficients of a polynomial are 0 , it is called the *zero polynomial* and is denoted by 0 . The zero polynomial is not assigned a degree.

Most of the terminology used for a polynomial in one variable carries over to the discussion of polynomials in several variables. But the *degree of a term* in a polynomial in several variables is obtained by adding the powers of all variables in the term, and the *degree of the polynomial* is given by the highest degree of all its terms. For example, the polynomial

$$2x^2y^5 - 3xy^3 + 8xy^2 - 3y + 4$$

is a polynomial in the two variables x and y . It has five terms with degrees $7, 4, 3, 1$, and 0 , respectively. Accordingly, the degree of the polynomial is 7 .

Adding and Subtracting Polynomials

Constant terms and terms that have the same variable and exponent are called *like* or *similar* terms. Like terms may be combined by adding or subtracting their numerical coefficients. For example,

$$3x + 7x = (3 + 7)x = 10x \quad \text{Add like terms.}$$

and

$$\frac{1}{2}m^2 - 3m^2 = \left(\frac{1}{2} - 3\right)m^2 = -\frac{5}{2}m^2 \quad \text{Subtract like terms.}$$

The distributive property of the real number system,

$$ab + ac = a(b + c)$$

is used to justify this procedure.

To add or subtract two or more polynomials, first remove the parentheses and then combine like terms. The resulting expression is then written in order of decreasing degree from left to right.

EXAMPLE 3

$$\begin{aligned} \text{a. } & (3x^3 + 2x^2 - 4x + 5) + (-2x^3 - 2x^2 - 2) \\ &= 3x^3 + 2x^2 - 4x + 5 - 2x^3 - 2x^2 - 2 && \text{Remove parentheses.} \\ &= 3x^3 - 2x^3 + 2x^2 - 2x^2 - 4x + 5 - 2 && \text{Group like terms together.} \\ &= x^3 - 4x + 3 && \text{Combine like terms.} \end{aligned}$$